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ABSTRACT

The paper describes evolution of an approach to teaching mathematically disabled and slow learning students through a Piagetian framework. It is explained that a step-by-step procedure is used to internalize material actions into mental actions via perception and verbalization. Formulae are introduced early, and emphasis is placed on promoting generalization. The issue of the choice of the material actions most suited for the subject to be taught is discussed, and the balance concept is favored over the traditional part-whole relationship. A final comment on the principle of mathematical variability illustrates the need to help students examine relations from various points of view. (CL)

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Illustration of a learning procedure

In our attempts to develop meaningful mental actions in mathematically disabled children and slow learners a learning procedure evolved gradually, the essential features of which we want to explain and to discuss.
Our theoretical point of departure is Piagetian, assuming that action upon concrete materials is internalized into mental actions. However, Piaget has little advice to give as to how this internalization process might be structured, expedited and actualized through teaching, Gal'perin, on the contrary, recommends the introduction of a step-by-step procedure, which has inspired us in many ways.
We shall give an impression of our procedure under five headings.

Our first point is the internalization process. How do we get from material actions to mental actions? (1)
Secondly we shall raise the matter of symbolization. How do we introduce mathematical formulae? (2)
Our next point is concerned with the "translation" processes, illustrating how we move from one material representation to other ones in order to stimulate generalization. (3)
These three points will only be dealt with shortly, just to give an idea of what we are aiming at. In doing so full attention can be paid to our crucial question: <u>which</u> material actions are most suited for each subject under consideration?
This question gives rise to a discussion of the general framework in which new subject-matters could be placed meaningfully and smoothly. (4)
Our fifth point is closely related to the previous one and deals with what we call mathematical variability.

1. In developing mental actions (M.A.) we start from a material basis and systematically, step by step, we internalize the material actions (Mt.A.).

Mt.A. ——————→ M.A.

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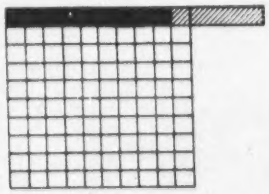
This is done through the incorporation into the learning procedure of two other forms of action between material and internalized mental actions:

a. perceptual action, relying on visualization and imagination, intended to detach the thinking process from the handling of concrete objects and their specific properties. As an illustration of perceptual action we take the sum 9+5:



The material action is putting e.g. a Cuisénaire rod of 9 and one of 5 on the 100-field, exchanging 5 for 1 and 4, putting the 4-rod in the second row and looking what the result is now: 10+4=14.

All of these material actions can be replaced by perceptual actions:

- (once the rods of 9 and of 5 have been put on the field) we can visualize splitting up 5, looking at the rods
- we can also visualize adding 5 when only the 9-rod is placed
- we can even imagine the 9-rod and visualize adding 5.

Through all these successive steps not only internalizing is realized but at the same time abbreviation takes place by skipping some steps and by combining others.

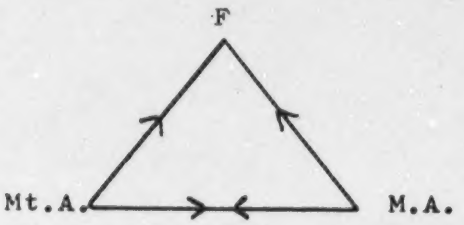
Very important is the gradual removal of the material apparatus, guaranteeing a lasting connection between the internalized action and its material basis, to which a child can return whenever he feels uncertain.

Mt.A. M.A.

b. verbalizing the actions to arrive at reflective thought.

2. In mathematical development special care should be given to the symbolization of actions, that is to the use of formulae (F).

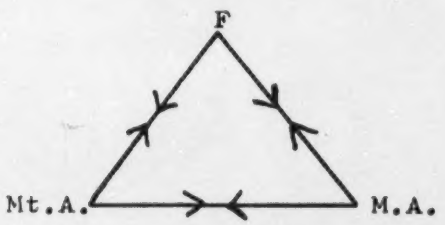
Therefore we introduce a third link into our chain:



The introduction of formulae takes place at a very early stage, during the period of mastering the material actions. This is done with a view of making pupils familiar with the idea that formulae literally represent the very actions which they stand for, with no hidden mystery whatsoever. Naturally we start with tiny formulae like "5-3" and we make sure that the pupils understand every possible connection between this formula and its different manifestations so that they can

- draw the action that goes with it: OØO ØØ
- materialize the action
- find the formula when the picture is shown
- find the formula when the concrete situation is given.

We call them "translations". The same problem (5-3) is represented in different "languages": spoken, drawn, written, materialized.
Thus we guarantee a strong and clear two-way link between each formula and the situation it expresses:

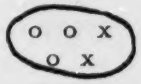


Under heading 4 we shall bring up the equal-sign, which needs a careful introduction. Many children consider 5-3=. and .-3=5 to be the same story and put a "2" in both cases without much hesitation, if any. This is the result of inadequate teaching, leading to insufficient, too peripheral connections between the symbols and what they stand for.

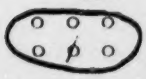
3. Our third point concerns liberating the mathematical processes and concepts from their material bases, this time aiming at generalizing and not at internalizing in the first place.
We introduce a variety of materials to make clear that the processes and concepts can appear in many concrete ways but exist on their own, independent of any casual materializing. Numbers e.g. can be materialized in a structured or unstructured way, with counters, Dienes blocks, money or other materials; we can find them on a one hundred square or on the numberline. Like the - intermodal - "translations" described under heading 2 we now "translate" from one kind of material to the other. These intramodal translations prevent fixation to one material basis and underline the casualness of each specific materializing and with that they emphasize the essence of the processes and concepts themselves.

4. We judge our fourth point to be of great importance: the question about the choice of the material actions that are most suited for the subject we want to teach. What are our criteria to decide what actions are suitable? At least they should not be isolated actions, but embedded in a general framework which focusses the attention on the essential features and the essential relations of the subject-matter. An example will make things more clear. We choose early arithmetic as the subject-matter.

For a long time we started from the traditionally and universally accepted basis, i.e. the part whole relationships, represented in this way:



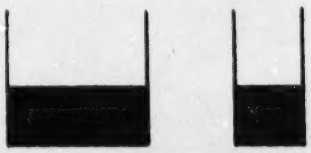
three and two make five, symbolized as 3+2=5



six minus one; five left over, symbolized as 6-1=5

The binary operation to the left is translated into the result on the right.
But on this basis we encountered serious problems.
Besides our own research much work on this topic is being done (v.Eerde, Verhoef; de Corte, Verschaffel; Ginsburg; Kieran) and it is becoming more and more clear to us that introducing and using the equal-sign as a symbol for the result of an operation leads to an operator notion of equality, the equal-sign becoming a kind of "do something"-signal. The operator notion of equality is a poor and a very restricted one. It is sufficient in sums like a $\pm$ b=, but does not work perfectly in open sentence problems such as a $\pm$. = b or . $\pm$ a = b, where it is much more difficult to identify the actions intended in the formulae.

In our view early arithmetic needs a new material basis and other material actions, that result in grasping the essential features of the equality relation. In this way a deeper notion of what the equal-sign stands for will evolve.
Inspired by the Russian psychologist Davydov we chose a new material basis, the balance, and developed new material actions, comparing, as an alternative for the part whole relations. The following will give an impression of our comparison programme.
First of all we make the pupils compare things on various measurable attributes like length, height, weight, volume, area etc., using the symbols =, > and < , so that they find out that things can e.g. be equal in height but yet different in volume:



Soon it becomes clear that before comparing things there should be an agreement as to which attribute is to be considered. So analysis is called for.
These two rows can be compared for different features.

A ● ● ● ● ● ● ● ● ●

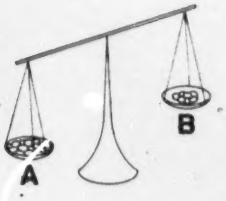
B ● ● ● ● ● ● ● ● ●

Considering length the upper row is greater: A > B.

Considering number the rows are equal: A = B.

Essentially this provides pupils with a clear idea of what "number" is about and - equally important - what "number" is not about.

After they have mastered comparing things we introduce a dynamic element and the pupils learn the operations needed for the transitions from equal to unequal and v.v.



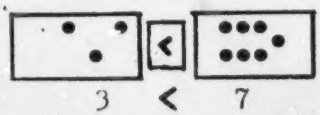
If A is the heavier one, where should I take off some peas if I want A and B to become equal in weight?

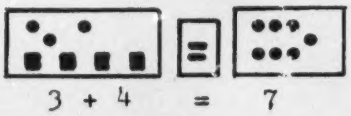
Those operations are not done by trial and error, but by careful predicting, followed by a trial using the peas themselves.

Step by step the pupils come to understand that the transition laws are general and fitting for each specific case and that these two lines ——— — can be taken as a general balance scheme which covers all eventualities.

The sums which the pupils will now be asked to do will be interpreted within this same theoretical framework, with the notion of comparison across the equal-sign as the fundamental concept.

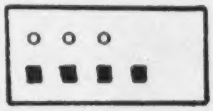
The difference between the two material bases of part whole relations and "balancing" is substantial. In a comparison both parts of the (in)equality are represented in the concrete material and the formula reflects the situation immediately and completely, eventual operations included:





In the part whole model, however, only one part of the equality is represented and the formula does not reflect the

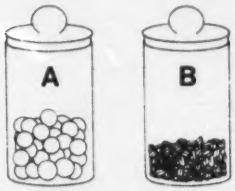
action as it does in comparisons:



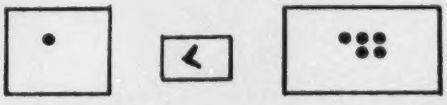
3 + 4 = 7

Of course children also need the part whole relation scheme e.g. as a framework for many word problems, but as this scheme seems to be more complex than the balance scheme and - in our opinion - less suited as the basis for early arithmetic, we first introduce the comparison programme and postpone the part whole relations to a later stage.

5. The principle of mathematical variability is specially aimed at making the pupils look at things and relations from various points of view. We shall illustrate this general principle by two comparison problems.
   a. The objects are two bottles, one filled up with glass marbles (A), the other one with peanuts (B):



   Instead of asking for the right sign (=, > or <) between the bottles:
   "Which is the greater one considering weight?"
   "Which is the greater one considering number?" and so on, we make the pupils look from other points of view. We ask them e.g.:
   "Which feature belongs to the comparison A > B?", or
   "Change A so that A > B is with respect to number".
   b. With two numbers of counters as objects we can ask:
   "Look at this comparison;



   can you restore it if I reverse the comparison sign?"

In this case there is an infinite number of correct solutions, together with a few incorrect ones. Children do not only enjoy this freedom to choose their own solution, but are also stimulated to approach problems in an active and reflective way.

References

Borghouts-van Erp, J.W.M., Rekenproblemen; opsporen en oplossen, Orthovisies 6, Wolters-Noordhoff, 1978.

Borghouts-van Erp, J.W.M., Spelen rond het Evenwicht (vert. Playing at Balancing), Stichting Film en Wetenschap, Hengeveldstraat 29, Utrecht, 1981.

Borghouts-van Erp, J.W.M., Clarity about the meaning of the equal-sign by "Playing at Balancing", Proceedings of the Sixth International Conference for the Psychology of Mathematics Education; Antwerp July 1982, 214-217.

Corte, E. de en L. Verschaffel, Kwalitatief psychologische analyse van het oplossen van aanvankelijke rekenopgaven bij 6 à 8 jarige basisschoolleerlingen, Pedagogische Studiën (57), 9, 1980.

Corte, E. de en L. Verschaffel, Een exploratief onderwijsexperiment met aanvankelijke rekenopgaven bij 6 à 8 jarige kinderen, Pedagogische Studiën (57), 10, 1980.

Davydov, V.V., De introductie van het begrip grootheid in de eerste klas van de basisschool, In: Van Parreren, C.F. en Nelissen, J.M.C., Rekenen; teksten en analyses, Sovjetpsychologie2, Wolters-Noordhoff, 1977.

Eerde, D. van en L. Verhoef, Kwantiwijzer, S.V.O.-project 0327, Psychologisch Laboratorium, R.U. Utrecht, Interimrapport III. 1979.

Ginsburg, H., Children's arithmetic; the learning process, New York, 1977.

Kieran, C., The interpretation of the equal-sign; symbol for an equivalence relation vs. an operator symbol; Proceedings of the Fourth International Conference for the Psychology of Mathematics Education, Berkeley, California; August 1980, 163-169.

Parreren, C.F. van, en Carpay, J.A.M.; Sovjetpsychologen over Onderwijs en Cognitieve Ontwikkeling, Wolters-Noordhoff, 1980.